A Simple Climate-Solow Model for Introducing the Economics of Climate Change to Undergraduate Students

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Abstract

In this paper we develop the simplest integrated assessment model in order to illustrate to undergraduate students the economic issues associated with climate change. The growth model developed in this paper is an extension of the Solow model and includes a simple climate model. Even though the model is very simple it is very powerful in its predictions. Students explore various scenarios illustrating how economic activity today will inflict damages on future generations. But students also observe that future generations will be richer than today's generation due to productivity growth and population stabilization. Hence, the richer future generations will not be as rich as they would be without climate change. Since the cost of action is absorbed by the current generation and the benefits of action accrue to future generations students can conduct a cost-benefit analysis and explore the importance of the discount rate.

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1. Introduction

"Greenhouse gas (GHG) emissions are externalities and represent the biggest market failure the world has seen" –Sir Nicholas Stern (2007)

Climate change caused by the greenhouse gas emissions released by the burning of fossil fuels and land use changes imposes damages to future generations.[2] GHG emissions trap heat and affect the future climate resulting in damages from increased temperatures. For example, increased temperatures are expected to cause sea level rise, increased floods, increased droughts and heat waves, and possibly even increased human conflict. The current generation benefits from using fossil fuels, but does not internalize these external costs. As a result, climate change is what economists call a negative externality. Without government intervention, humans will overproduce greenhouse gas emissions.

The climate change problem is further complicated as being a global externality rather than a local one. Even though each nation emits a different amount of GHG, the marginal impact of a tonne of GHG is independent of where it is emitted (Stern, 2007); whereas, the effects of smog in a city are local and heterogeneous depending on the geography and demographics of the city. Furthermore, GHGs accumulate in the atmosphere and stay a long time, i.e., carbon dioxide has an average atmospheric life of over a century (Archer et al., 2013). The impact is persistent and long term, whereas the effects of smog in a city are relatively immediate following exposure.

Due to the persistence of GHGs in the atmosphere, the climate change problem is characterized by the issue of inter-generational equity: The current generation is imposing external costs on future generations and would have to forego some economic growth to limit those costs. But at the same time, it is also characterized by issues of intra-generational equity,

[2] For scientific consensus on the issue see Oreskes, (2007).

for example, rich nations which are relatively GHG intensive are located in temperate climates and have the funds and strong institutions to more easily adapt to climate change; whereas, poorer nations, say in sub-Saharan Africa, are expected to be hit relatively harder by the negative impacts of higher temperatures.[3]

Complicating the problem is the fact that uncertainty and risk are significant. Damages from climate change could be potentially large and irreversible even though the probability is relatively low for such extreme events (Weitzman, 2009, 2011).

Furthermore, the atmosphere, the oceans, and public lands share the characteristics of a common pool resource in that they are rivalrous and no one is excluded from using these to dump emissions. The continuous disposal of carbon into the atmosphere, oceans, and land could thus result in the tragedy of the commons (Broome, 2012).

Finally, reducing GHG emissions can be characterized as a public good in that the benefits of mitigation are non-rival and non-exclusionary. No one can be excluded from enjoying a more stable climate which results in a free-rider problem and the under-provision of mitigation policy. [4] It is no wonder that Sir Nicholas Stern considers this issue the biggest market failure the world has ever seen.

In spite of these significant issues and all the research being undertaken to study the economics of climate change, not much has been formally done to introduce this topic to undergraduate students. Tol (2014) is a notable exception, as a text on climate economics suitable for a full course in climate economics with a specific focus on integrated assessment modelling (IAM) at the masters' or advanced undergraduate levels. Yet there is little available to introduce undergraduate students to IAMs for a one to two week portion of a climate economics

[3] For a critical review of inter-generational and intra-generational climate justice see Forsyth (2013).
[4] Recently, Nordhaus (2015) has proposed the formation of climate clubs to solve the free rider problem.

or environmental economics course. We find the existing IAMs (DICE, FUND, and PAGE) overly complex for teaching the economics of climate change to undergraduate students. For example, the DICE model is based on the Ramsey growth model that many economics students do not encounter until graduate school. Our approach adjusts the Solow growth model that undergraduate economics students are familiar with. Furthermore, the existing IAMs include a complex representation of the climate system that takes a significant amount of time to explain to undergraduate students. Our approach uses a simple linear relationship between atmospheric carbon accumulation and expected temperature change demonstrated by Matthews et al (2012). Our paper is aimed to explain to instructors the usefulness of our model (contained in an accompanying Excel workbook) for introducing undergraduate students to the economics of climate change. Throughout the paper we provide figures and key points for instructors to highlight to students and to use as starting points to motivate in-class discussion.

The closest works to our model are Nordhaus' DICE model, Brock and Taylor (2010), and Taylor (2014). Unlike the DICE model, we use the Solow growth model and a linear climate system. Unlike Brock and Taylor (2010) and Taylor (2014), we include damages from increased temperatures. Taylor (2014) is also based on the Solow model and uses a similar linear relationship between carbon accumulation and temperature change, but does not take the next step of including damages from temperature change back into the growth model. Yet, none of these three closely related works are aimed at educating undergraduate students about the economics of climate change.[5]

Section 2 described the basic climate-Solow-Model for the world economy. The presentation of the simple model follows the same way it would be presented to students who

[5] Even though Pindyck (2013, 2015) considers IAMs useless to guide policy such models have been used consistently by various researchers to illustrate issues associated with the economics of climate change.

have already been exposed to the basic Solow model. In case students are not exposed to the Solow model more time can be spent explaining the basics of the Solow model and the concept of steady state levels. Furthermore, students unfamiliar with the Solow model will need to understand how changes in the savings rate, population growth rate, productivity growth amongst others affect steady state variables such as the standard of living as measured by income per person. Section 3 alters the model to examine damages which are more severe and considered in the academic literature on the economics of climate change. This section illustrates to students the impact of climate change when temperature increases affect the depreciation of capital and productivity growth. In addition, students can use Weitzman's damage function to explore more severe damages when temperature increases exceed 4 degress Celsius. Section 4 uses the model developed in section 2 to illustrate to students' the cost and benefits of emission reductions by conducting a simple Benefit-Cost Analysis for the 2 degrees target. This analysis allows students to understand the important role of the discount rate in evaluating climate policy. Finally, concluding remarks and other possible classroom extensions are mentioned. The Appendix provides detailed information on the parameters used and setting up the excel spreadsheet for the base case.

2. The Simple Climate-Solow Model

2.1 Economic Growth & Climate Impacts

The economic growth component of the model is a variation on the standard Solow Growth model. In the standard undergraduate treatment of the Solow model, output is produced by the combination of capital, K_t labor, L_t and technology, A_t according to the Cobb-Douglas production function $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$, which can be rearranged in terms of output per worker as

$$y_t = A_t k_t^{\alpha}.$$

This is the standard Solow Growth model that students should be already familiar with. For the purposes of studying climate change, the effect of increased temperatures is added to the model in a similar way as by Nordhaus (2008) and Fankhauser and Tol (2005). The production function in our model is slightly altered to be the following

$$y_t = D_t A_t \, k_t^{\alpha},$$

where $D_t = 1/(1 + \theta_1 T_t^{\theta_2}) \ \leq 1$ is the damage function and T_t is the temperature anomaly in year t. The production function looks the same as the standard Cobb-Douglas production function, except output per worker is now reduced by increased temperatures, i.e., the higher is T_t, the lower is y_t *ceteris paribus*.

The savings rate, s is constant, leading to investment per worker in period t of sy_t. Capital depreciates at a constant rate, δ_K. To reflect recent UN population projections that predict global population will plateau around 10.5 billion, total population and the labor force grow at a decreasing rate over time, $g_{L,t} = g_{L,0}/(1 + \delta_L)^t$ determined by the parameter $\delta_L > 0$ which reduces the degree of population growth over time. The term $g_{L,0}$ is the population growth rate in the base year of 2010. Total factor productivity, A_t also grows at a decreasing rate over time: $g_{A,t} = g_{A,0}/(1 + \delta_A)^t$.[6] This leads to the following difference equation to describe the transitional dynamics in the model:

$$k_{t+1} - k_t = sy_t - \left(\delta_K + g_{L,t}\right) k_t.$$

[6] The assumption of a declining growth rates of total factor productivity and population growth as shown above are also used in Nordhaus (2013). Most undergraduate students will be familiar with the Solow model with constant rates of population and technology growth; therefore, the diminishing growth rates used here may appear more complicated at first glance to the students. However, this change has little effect on how an instructor would traditionally introduce the dynamics of the Solow model.

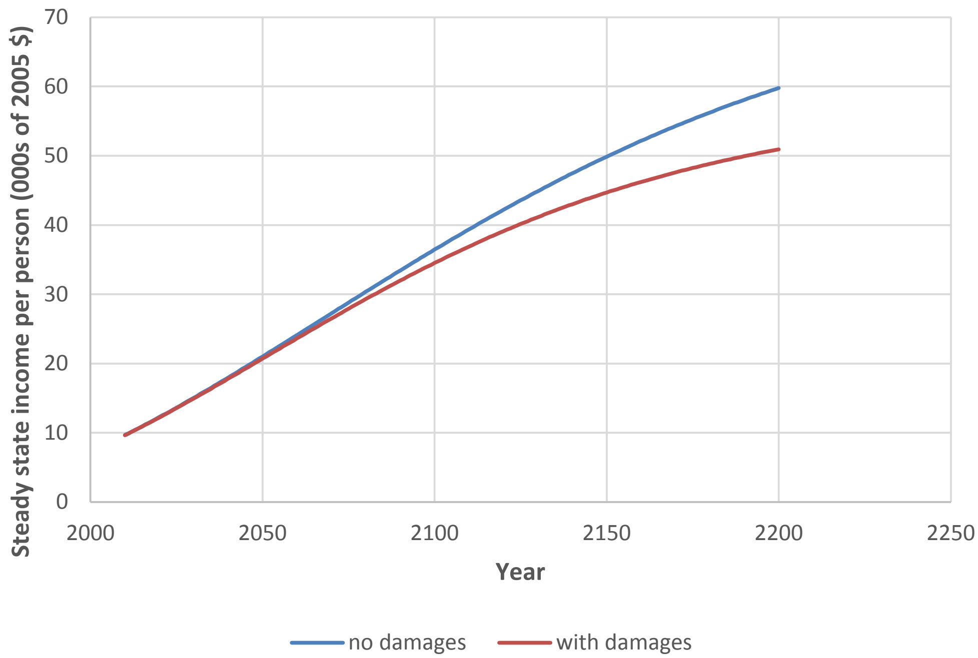
Given this equation it is easy to show convergence to a balanced growth stable steady state capital labour ratio $k_{ss,t} = \left[\frac{sA_tD_t}{\delta_K + g_{n,t}}\right]^{1/(1-\alpha)}$ for a given time period t.[7] Due to population growth declining and technology advancing, the balanced growth steady state capital labor ratio will increase over time (offset by damages). Along the balanced growth path, output per worker, $y_{ss,t} = D_t A_t \, k_{ss,t}^{\alpha}$ grows at a rate dependent on changes in temperature (outlined in subsection 2.3), the growth rate of total factor productivity, $g_{A,t}$ (which grows at a declining rate) and the growth rate of the capital labour ratio which is weighted by the income share of capital, α. It can be easily seen that in the absence of climate damages (i.e., $D_t = 1$), y_t grows at a faster rate.

To identify the impact of Business-As-Usual (BAU) in the model, we can provide a simple comparison of $D_t = 1$ for all *t* (i.e., no climate damages) to $D_t < 1$ (i.e., with climate damages). This comparison is shown in Figure 1 for the parameter values displayed in the appendix. The figure is very useful to highlight to students the central trade-off involved in the climate change problem. There are two important aspects of this figure to highlight. First, that the model, consistent with other IAMs, predicts that future generations are better off despite climate damages. Second, that the climate change problem is intergenerational in nature; the damages of climate change, as represented by the wedge between the two lines, are imposed mainly on future generations. Combined, these two aspects highlight that the climate change problem can be encapsulated by the following trade-off: A relatively poorer current generation is imposing damages (costs) on relatively richer future generations. This is of course only true in the base case of the model, and altering either the damage function or where damages enter the

[7] Simulations can also conducted using transitional dynamics but we leave this as an extension. The differences between the two paths is not significant and this path will converge to the same unique steady state values when technology is constant and population growth is constant. We use the balanced growth path.

model can lead to future generations being made worse off; which is a useful exercise for instructors to do for their class using our provided Excel workbook.

Figure 1: The Solow Model with and without Climate Impacts



Source: Authors' calculations.

Classroom discussion: Future generations are worse off because of climate change but future generations will be richer than the current generation. What are the implications?

2.2 Carbon Emissions

Carbon emissions are generated in our model by the production process based on the assumed emissions intensity (emissions per unit of output) at time *t*. Emissions intensity is given by the following formula

$$\sigma_t = \frac{E_t}{Y_t},$$

where E_t is tonnes of carbon released and Y_t is total output. For modelling purposes, we take the approach of assuming a level of emissions intensity in the base year and then specifying the growth rate of emissions intensity over time into the future. Figure 2 shows that global emissions intensity has steadily declined between 1950 and 2010. We assume that future declines in emissions intensity take the following relationship

$$g_{\sigma,t} = g_{\sigma,t-1}/(1 + \delta_\sigma),$$

where $g_{\sigma,t} < 0$ is the growth rate of emissions intensity between periods t and t-1 and $\delta_\sigma < 0$. The value of emissions intensity in year t can then be calculated as[8]

$$\sigma_t = \sigma_{t-1}\left(1 + g_{\sigma,t}\right).$$

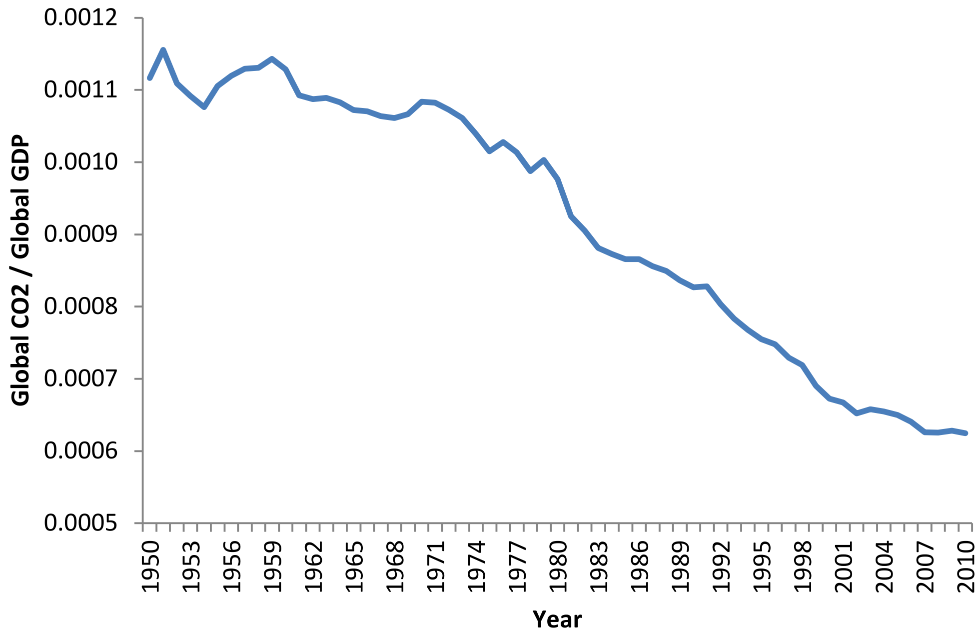
This formula can also be expressed in terms of the base year

$$\sigma_t = \sigma_0 \prod_{i=1}^{n=t} \left[1 + g_{a,0}/(1 + \delta_\sigma)^i\right].$$

This information is provided for the benefit of instructors and can be given to especially interested students; however, the important thing to highlight to students is that emissions intensity of output is assumed to decline at an increasing rate into the future (consistent with past history).

[8] Similar assumptions about emissions intensity were made by Nordhaus (2013). For details see http://www.econ.yale.edu/~nordhaus/homepage/documents/DICE_Manual_103113r2.pdf

Figure 2. Global Emissions Intensity, 1950-2010



Source: CDIAC, 2015; Maddison Project, 2013; authors' calculations.

Carbon emissions in year t are then calculated by multiplying the emissions intensity in year *t* by the output in year *t*

$$E_t = \sigma_t Y_t.$$

The carbon emissions predicted by the model follow an inverse-u shape consistent with the Environmental Kuznets' Curve hypothesis and are displayed in Figure 3A and 3B. As income per capita increases emissions initially increase, peak in the later part of this century when income per capita reaches thirty thousand dollars and then emissions start declining. Along a steady state, emissions initially grow because output grows faster than the rate at which intensity falls but after a certain period the latter becomes stronger than the former causing emissions to fall. This can be seen as follows (See also Taylor and Brock for a similar expression, 2010):

$$g_{E,t} = g_{\sigma,t} + g_{Y,t}.$$

This relationship is important as it indicates to students how difficult it is to reduce emissions in an economy that is growing along a steady state due to population growth, total factor productivity growth and capital per worker growth.[9]

This relationship can also be connected to the IPAT equation when expressed in growth rates. The IPAT equation is used by the IPCC for setting future emission targets. It links environmental impact (I) to population (P), affluence (A) and technology (T). In our experience, students find the IPAT equation easy to understand even though it is an identity.[10] The IPAT equation for carbon emissions is usually expressed as follows:

$$E_t \equiv P_t \frac{Y_t}{P_t} \frac{E_t}{Y_t}.$$

Carbon dioxide emissions at time t (i.e., E_t) are proportional to population multiplied by affluence as measured by output per capita at time t and technology as measured by carbon emissions per dollar of output (recall that in our model $E_t / Y_t = \sigma_t$). In growth rates, after cancelling out the growth of population, this identity becomes:

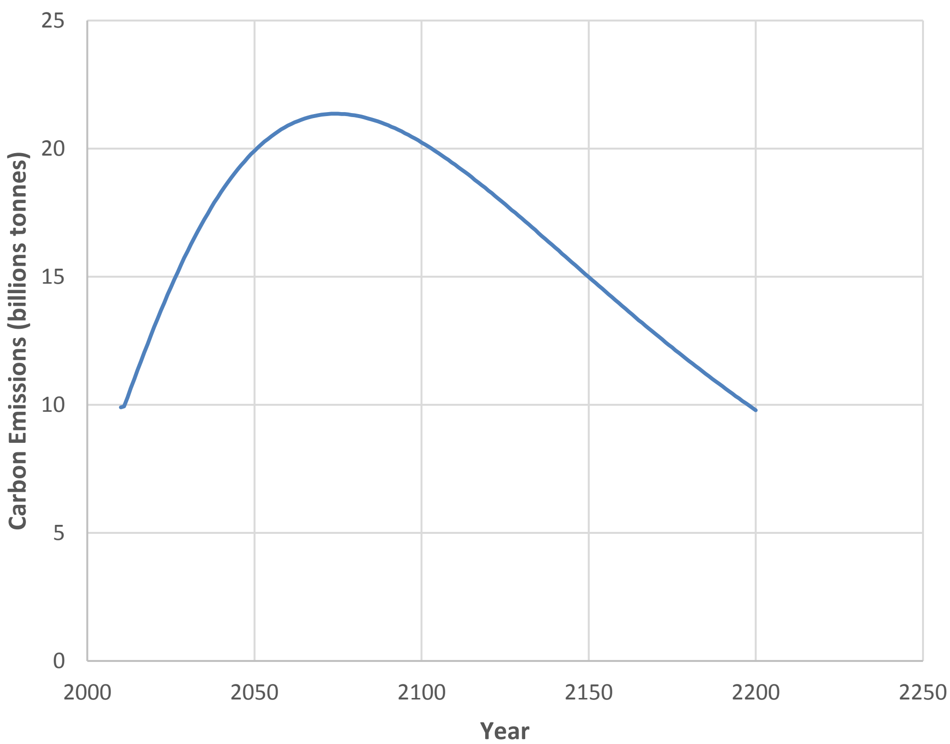
$$g_{E,t} \equiv g_{Y,t} + g_{\sigma,t},$$

which is identical to the growth rate of emissions from our model. The difference now is that the Solow model provides a theory that explains why output grows. Emissions grow because affluence grows along a steady state that in the Solow model is due to population growth, growth in total factor productivity and growth of capital per worker offset by the impact on growth from damages growing over time. The emissions growth rate is also affected by the emissions intensity falling over time (i.e., $g_{\sigma,t} < 0$). Hence the IPAT equation in growth rates arises from the long run properties of the Solow model and can explain why our model produces an inverse

[9] This is offset partially by the growth rate of the damage that occurs with increasing temperature.
[10] We have students downloading yearly data from Gapminder.org to explore this relationship for individual countries.

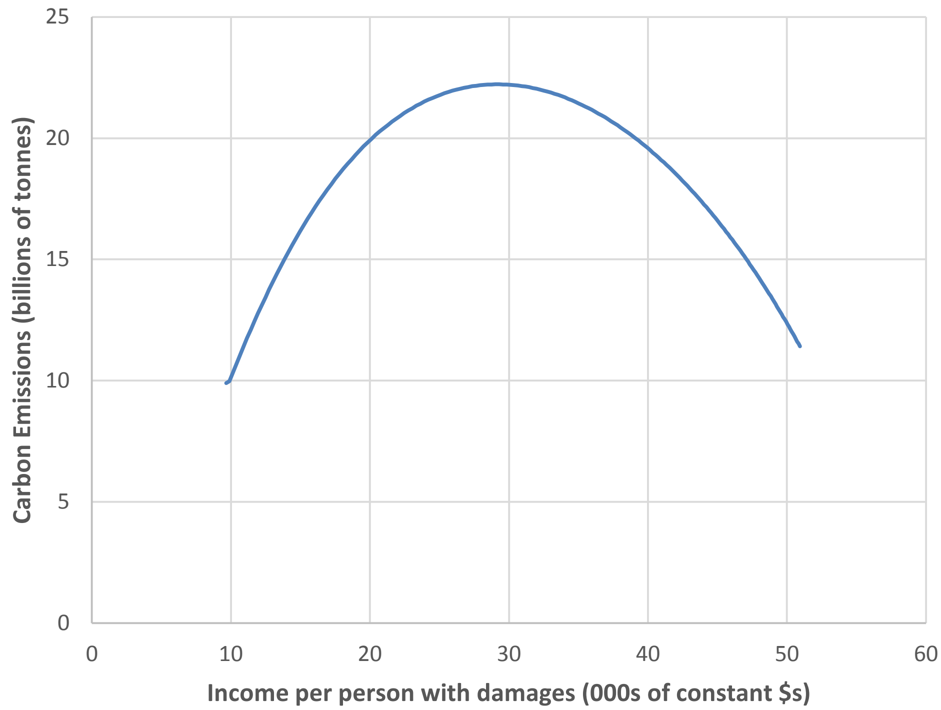
u-shaped emissions path over time (as displayed in Figure 3A). At first, $-g_{\sigma,t} < g_{Y,t}$ but over time the growth rate of output slows down (since we assumed diminishing TFP and population growth) and eventually $-g_{\sigma,t} > g_{Y,t}$ producing negative emissions growth (i.e., $g_{E,t} < 0$).

Figure 3A. Predicted Global Carbon Emissions, 2010-2200



Source: Authors' calculations.

Figure 3B. Environmental Kuznet's Curve



Classroom exercise and discussion: Ask students to use the IPAT equation

$$E_t \equiv P_t \frac{Y_t}{P_t}\frac{E_t}{Y_t}$$

to find what it takes in terms of technology to reduce emissions in 2050 by 50% below 2010 levels with an assumed population growth of 1.5 percent and growth of affluence as measured by income per person by 2.5% per year.

2.3 Carbon Accumulation & Temperature Change

One of the aspects that make this model so useful for teaching is the simplicity of how the climate system is modelled.[11] We make use of the simple proportional stable linear relationship

[11] It is important to give students a basic understanding of the science of climate change before exposing them to the modelling of temperature anomaly. Basics understanding of climate change can be found at the U.S. EPA

between carbon accumulation and global temperature change found by Matthews et al. (2012). They found that temperature increases by approximately 1.8 Celsius per 1000 billion tonnes of carbon (i.e., 1000 PgC) emitted with a 95 percent confidence band between 1 and 2.5 degrees Celsius. This relationship is found to be independent of both time and the level of stabilization of atmospheric carbon concentration (i.e., the emissions scenario). Using this scientifically based relationship avoids modelling much of the complexity of the climate system done by other IAM models.[12] The following relationship shows the cumulative emissions from pre-industrial levels to 2010. We define the cumulative emissions from the pre-industrial levels to 2010 (the base year for our simulations) as C_0, i.e., these are the sum of past emission releases. The global temperature change relationship to carbon accumulation into the future is:

$$T_t = \beta \left[C_0 + \sum_{i=1}^{t} E_i \right],$$

where $t \geq 1$. The first term, $\beta\, C_0$, is the impact on global temperature change relative to pre-industrial levels due to the accumulated carbon emissions that were released prior to 2010 (i.e., there are 530 billion tonnes already accumulated). The second term, $\beta\, \sum_{i=1}^{t} E_i$, is the impact on global temperature at any time t in the future due to the additional emissions accumulated since 2010. Because of a growing economy, as shown in the previous section, emissions will continue to accumulate resulting in a higher temperature change.

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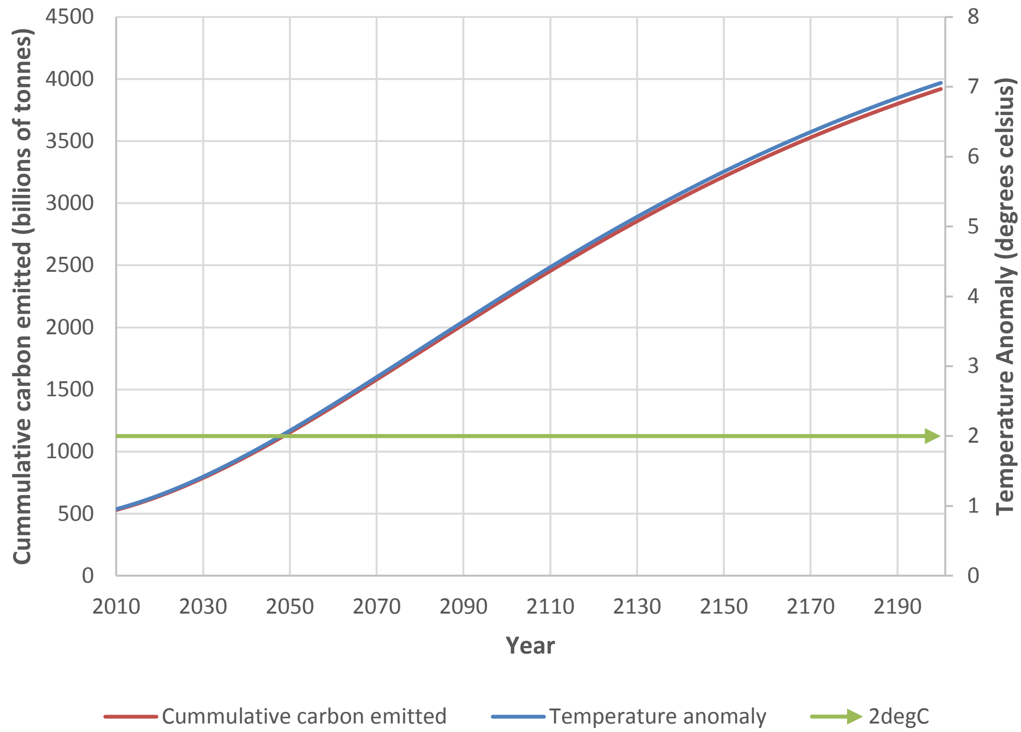
http://www.epa.gov/climatechange/basics/ or showing students the IPCC AR5 short video on the physical science basis at https://www.youtube.com/watch?v=6yiTZm0y1YA. For students that want to go beyond the basics on the science of climate change we recommend Professor Archer's video lectures or online course at: http://forecast.uchicago.edu/lectures.html.

[12] This complexity includes the uncertainty associated with the path of carbon emissions towards affecting the atmospheric concentration level, through carbon sensitivity, and then the impact of the concentration level of carbon to temperature anomaly change via the climate sensitivity parameter. A major part of uncertainty is associated with the climate-carbon feedbacks that take place over space and time.

Note that the above relationship is independent of the emissions pathway selected. What matters in terms of temperature change anomaly is the cumulative carbon emissions and the targeted budget. For example, if we are to keep global temperature anomaly below 2 degrees Celsius relative to pre-industrial levels then cumulative emissions should not increase more than approximately 1110 billion tonnes (i.e., the budget). If they increase by 470 billion tonnes over the next 50 years which is within the current BAU pathway (See Figure 3A and 3B) we will reach 1000 billion tonnes. This will result in a temperature increase of 1.8 degrees Celsius relative to pre-industrial level given that Matthew et al. found β to be 0.0018 per 1 billion tonnes of cumulative carbon emitted. Figure 4 shows the path of cumulative carbon emissions starting from 530 billion tonnes. Figure 4 also shows the corresponding temperature increases as well as the 2degC target. With business as usual we will hit 2 degrees Celsius just before 2050 and surpass 2000 billion tonnes by 2100 leading to a temperature increase of 3.6 degrees Celsius which is considered dangerous climate change.[13]

[13] There is an estimated 6000 PgC that can be accumulated given the fossil fuels available. Recently, the relationship has been found to be stable within 5000 PgC (Tokarska et al. 2015).

Figure 4. Predicted Cumulative Carbon Emissions and temperature anomaly



Source: authors' calculations.

Classroom exercise and discussion: Ask students to find different paths, using Figure 3A, in order to keep the accumulation of carbon below 1000 billion tones by 2100. Note that each box in Figure 3A is 250 billion tonnes of carbon and that we have already accumulated 530 billion tonnes since 2010. Can emissions increase in the short run? Does stabilizing emissions reduce the concentration? What are the implications of the alternative paths?

4 Additions to the base model for discussion

Damages in the base model enter multiplicatively in the production function as in Nordhaus (2013). It is assumed that climate change causes losses to production in the same period only via the damage function. Temperature increases are assumed not to affect the depreciation of physical capital nor any other form of capital such as environmental, social and organizational capital. In addition, climate change is assumed not to impact the factors of production individually nor the growth rate of total factor productivity. Also, the damage function used in the base model has been calibrated for losses when temperature increases to 2.5-3 degrees Celsius but it does not apply for higher temperature changes which are a real possibility under BAU (Stern, 2013). Furthermore, catastrophic damages are not incorporated into the base model (See Pyndick (2013), Weitzman (2013)). Below we incorporate some of these additions from the economics of climate change literature to the base model. This enriches the simple model in terms illustrating impacts to students. [14]

First consider the depreciation rate of physical capital. It is easy to conceive that increased temperatures and more severe weather will lead to capital having a shorter life span. It was mentioned as a possibility by Fankhauser and Tol (2005) and by Stern (2013). Recently, it has been incorporated into the DICE model by Dietz and Stern (2014) as well as Moore and Diaz (2015). Climate change can affect the durability and the longevity of stock of capital, for example, increased temperatures cause increased frequency of storms, more extreme weather, rising sea levels, and many other impacts. Such events can cause permanent damage to capital infrastructure. [15] Capital will require more maintenance to keep it from further wear and tear due to temperature rising. Capital could even be stranded if people move far away from the ocean shores due to sea level rising. Extremely powerful storms could destroy capital which then needs replacement. With temperature increasing, a larger fraction of investment spending will be allocated towards depreciation (and to adaptation measures) than to new investment which is the

[14] Stern (2013) suggests four alterations to the basic model to make it more relevant. First, damages to social, organizational and environmental capital. Secondly damages to the stock of capital and land. Third, damages to overall factor productivity. Finally, damages to learning and endogenous growth.

[15] Stern states: “Climate events such as storms or inundation can do permanent or long term damages to capital and land. If it is necessary to abandon certain areas, capital, infrastructure and land have zero use value and are essentially lost. This could be incorporated via a permanent damage or a reduction in capital occurring in period t as a result of temperature and events in that period.” pg. 849.

engine of economic growth. This increased spending on necessary investment, to keep the capital labour ratio constant, reduces the steady state capital per person and hence the steady state income per capita. A simple way to introduce the impact of temperature on the depreciation rate is as follows:[16]

$$\delta_K = \delta_0 + \delta_1 T_t.$$

For the simulations we suggest that the depreciation rate increase by 1 percent per 1degC temperature increase (i.e., δ_1=0.01). Currently, on average, capital is replaced after 10 years assuming the depreciation rate is at the base rate of 0.1. If temperature increases to 2degC (5degC) then capital will need to be replaced on average every 8.3 years (6.7 years) as it wears out faster.

Second, temperature increases could affect the growth rate of total factor productivity (Moyer et al, (2013), Dietz and Stern (2014), Moore and Diaz, (2015)). The growth rate of total factor productivity could be impacted negatively because resources will be diverted away from R&D and instead used for climate adaptation and for the reconstruction of capital due to climate damages. Furthermore, output per hour of input (labour or capital) could decline if inputs need more hours to produce the same output level due to a different climate environment. Also, a warmer climate will increase the likelihood of human conflict (Hsiang et al. (2013)). This in turn negatively impacts the institutions that protect property rights causing a possible reduction in the growth rate of total factor productivity. There is also evidence that economic growth is lower with higher temperatures (Dell, et. al. (2012)). We can model total factor productivity along the following lines:[17]

$$g_{A,t} = \frac{g_{A,0}}{(1+\delta_A)^t} - \gamma T_t.$$

[16] Stern (2013) suggested an equation along these lines (See page 850).
[17] This formulation is similar to Dell et al. (2013).

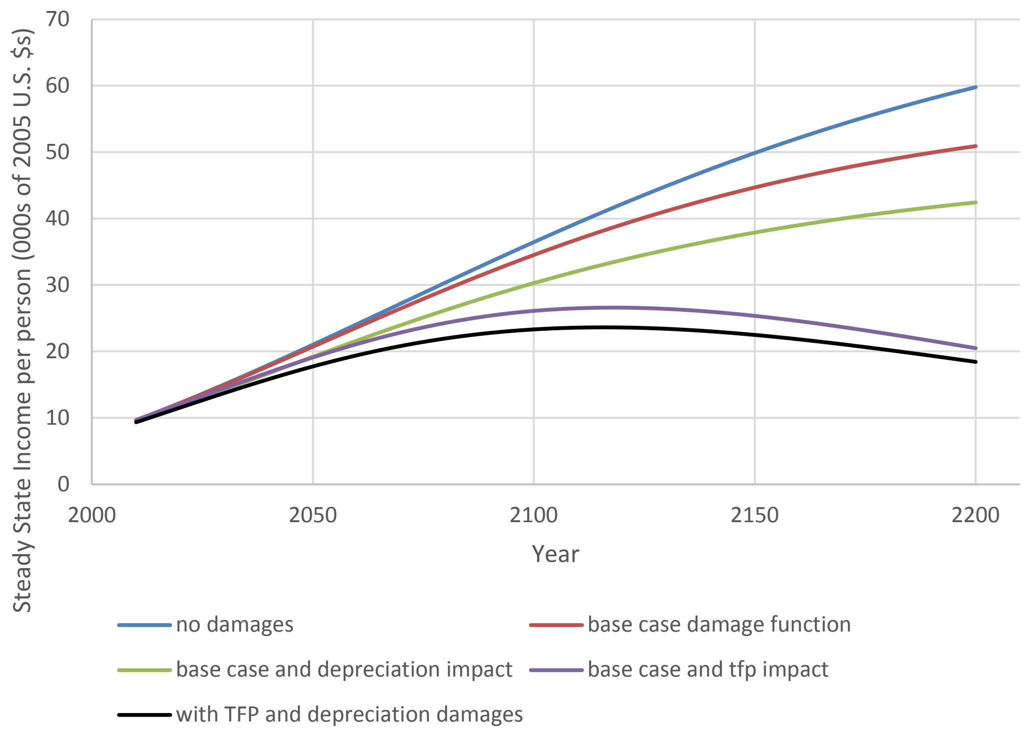
For the simulations we set $\gamma = 0.001$. It reduces the growth rate of total factor productivity by 0.001 for every 1 degree Celsius increase. Although this might seem like a small effect, it accumulates over time into a significant impact as it directly affects current production and the future growth rate of output per person. Note that total factor productivity is assumed to decline over time even without climate change but remains positive.[18] But with temperature rising beyond 3 degrees Celsius, the impact from the warmer temperature can offset the exogenous growth rate of total factor productivity.

Figure 5 shows the path of steady state output under base case damages (red path), base case and depreciation impact (green path), base case and impact on the growth rate of total factor productivity (purple path), and the black path under all damages. It is clear that adding only the depreciation effect does not cause a significant reduction in steady state income per person. Income per person is rising but starts at a lower level and remains lower relative to the base case. Steady state income per person increases to \$42,000 rather than to \$50,000. Adding the impact of climate change on the growth rate of total factor productivity amplifies the damages relative to the base case. In this case, steady state income increases initially, reaches a maximum after 2100, and then starts falling. Given that the growth rate of total factor productivity is assumed to decline even without climate change eventually there will be a temperature level after which the growth rate of total factor productivity will be negative and this will cause the reduction in steady state output per person. As seen from the path, the losses to output per person are significant if temperature increases affect the growth rate of total factor productivity. Taken all together the damages approach 80 percent relative to no climate change and almost 50 percent lower than the income per person observed in 2100 at the peak. Still income per person is higher,

[18] Dietz and Stern argued that this is due to depreciation of productivity (i.e., displacement of skills and know how) being stronger than institutional innovations that promote growth in productivity.

at approximately $18,400, than the current income level of approximately $10,000 (i.e., 80 percent higher). Note that the slower growth in output per person and the decline in income per person from 2100 helps in slowing down emissions and hence temperature increases. The temperature increases to 4.9 degrees Celsius by 2200.

Figure 5: The Solow Model with climate impacts on depreciation and TFP growth



Source: authors' calculations.

Another extension that can be incorporated into the simple model is associated with the type of damage function. The standard damage function in the DICE model represented in our base model has parameter values that have been calibrated for temperature increases not exceeding 3 degrees Celsius relative to pre-industrial levels.[19] Higher temperature levels than 3 Celsius are highly likely with BAU as seen from Figure 4. Figure 1 shows very low damages at

[19] More recently Nordhaus (2008, 2013) has altered the damage function to be $D_t = 1/(1 + \pi_1 T_t + \pi_2 T_t^2)$ but calibrated the parameters to yield similar results to the damage function used in the base model which was the Nordhaus, (1994) damage function.

temperatures which reach 6-7 degrees Celsius by 2200 with BAU. But such changes in the climate have not been observed for millions of years and could have profound impacts on the planet. Weitzman (2012) states that

> Six degrees of extra warming is about the upper limit of what the human mind can envision for how the state of the planet might change…and a temperature change of approximately 12 Celsius therefore represents an extreme threat to human civilization as we know it, even if it does not necessary mean the end of homo sapiens as a species. (pg. xx)

He envisions an increase of 18 degrees Celsius as the 'death temperature." Weitzman calibrates θ_1 to be 0.00238 so that it conforms to the Nordhaus (2008) DICE model. Nordhaus' model results in an eight percent loss from a temperature rise to 6 Celsius relative to pre-industrial levels and only a 26 percent loss with temperature increasing to 12 degrees. According to Weitzman this type of temperature increase (12 Celsius) was observed during the Eocene epoch, 55-34 million years ago with an ice free planet and alligators living near the North Pole. While for low temperature increases the damages are not high, and the Nordhaus parameters are fine, he argues that temperature increases of 6 Celsius can result in at least a 50 percent loss of output and that a 12 Celsius increase will result in a 99 percent loss of output. He recommends to capture this with the damage function along the following lines:

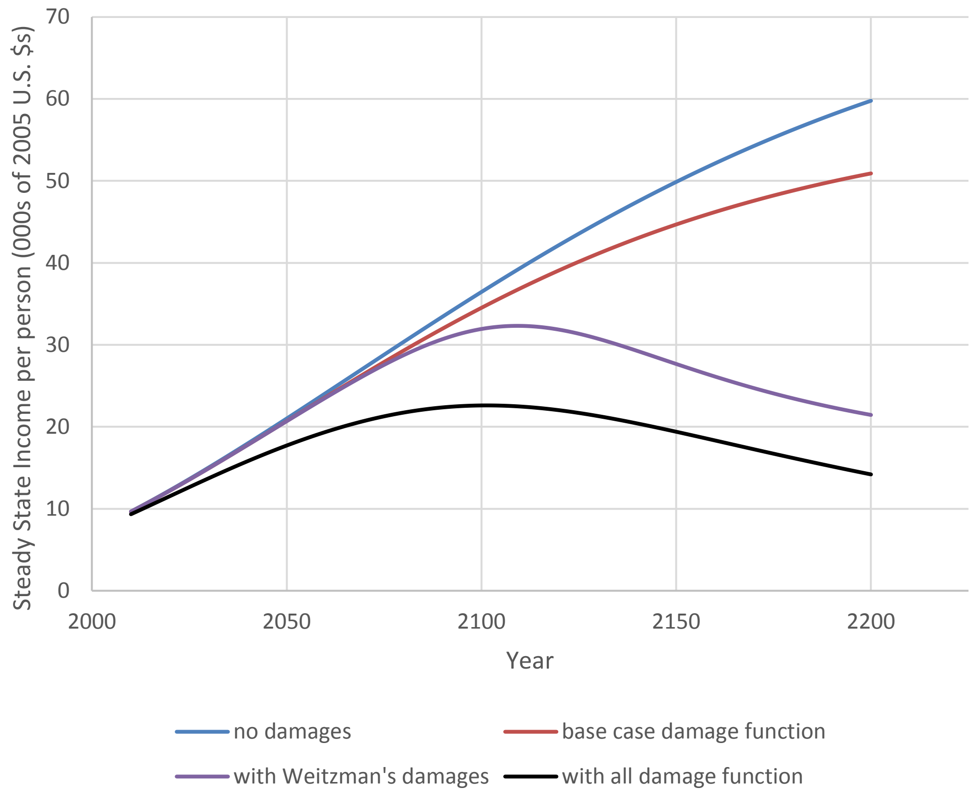
$$D_t = 1/(1 + \theta_1 T_t^{\theta_1} + \theta_3 T_t^{\theta_4})$$

where the parameters assigned to $\theta_1$and θ_2 are as the Nordhaus damage function but θ_3=0.507E-05 and θ_4= 6.754 as given in Table 1. A simple way to show students the fundamental difference between the Nordhaus and Weitzman damage functions is to use a traditional, static marginal damage graph with marginal damages on the vertical axis and emissions on the horizontal axis. Environmental economics students will be readily familiar with this type of graph. In this space, the Nordhaus marginal damage curve is linear and upwards sloping, whereas the Weitzman marginal damage curve is convex.

Figure 6 illustrates the different cases in our model. The path without climate change is the blue trajectory, followed by the red path with Nordhaus type of damages, followed by Weitzman's path (purple path) which closely follows the Nordhaus path until temperature increases reach 3 Celsius and then the last term in the damage function takes on a more important role and the two income paths start diverging. This can be considered a tipping point. When temperature reaches 4degC steady state income per person starts declining reaching $21,000 by 2200. Note that the current generation is poorer than the generation living in 2200 but the 2200 generation is poorer than a generation living around the year 2100.[20] Adding in possible damage impacts on the depreciation rate and the growth rate of TFP, δ_1=0.01 and $\gamma = 0.001$, the steady state income per person follows the black path. Steady state income per capita increases initially reaches a maximum of approximately $20,000 and then drops. This drop is due to the new damage function and the impact climate change has on the growth rate of total factor productivity. Steady state income per person declines and reaches $14,000 by 2200.

[20] With different parameter values it could be possible that the current generation is richer than one living in 2200.

Figure 6: The Solow Model with expanded climate impacts and new damage function



Source: authors' calculations.

Classroom discussion: What would the consequences be if damages negatively affected the growth rate of population?

Classroom exercise: Using the instructions from the Appendix ask students to reproduce the results in table 2 and to conduct a sensitivity analysis by changing the parameters.

Classroom discussion: What would the economic consequences be if there was a one-time impact arising from climate change such as a sudden destruction of the capital stock and/or population due to a tipping point occurring after 4 degrees Celsius? Would the steady state path be relevant to follow or would the transitional dynamic path be more appropriate in this case?

Classroom exercise: Using the transitional dynamic path in lieu of the balanced growth steady state path reproduce the income per person path when all damages are present. Can the future generation be worse off than the current generation?

4 Emissions Mitigation, Benefit-Cost Analysis & the 2 Degree Target

At COP16 in Cancun in 2010, attending nations agreed to a long-term goal of limiting the increase in average global temperatures to below 2 degrees Celsius (UNFCCC, 2011). In a recent meeting, the leaders of the G7 countries reiterated their commitment to this target (Carrel and Martin, 2015). To achieve this goal in our model, global cumulative carbon emissions should be limited to 1.111 trillion tons. Given that 530 billion tons have already been emitted by 2010, this leaves a carbon budget from 2010 on of 581 billion tons. If this budget is exceeded, temperature will have more than a 50% chance of exceeding 2 degrees.

Our simple model can be used to teach students about the costs and the benefits of the proposed target relative to business-as-usual as well as the importance of the choice of the discount rate for evaluating climate policy. Our model is very well suited to incorporating the 2 degree target and its carbon budget because temperature change is based directly on cumulative emissions. Capping cumulative emissions is therefore relatively straightforward.

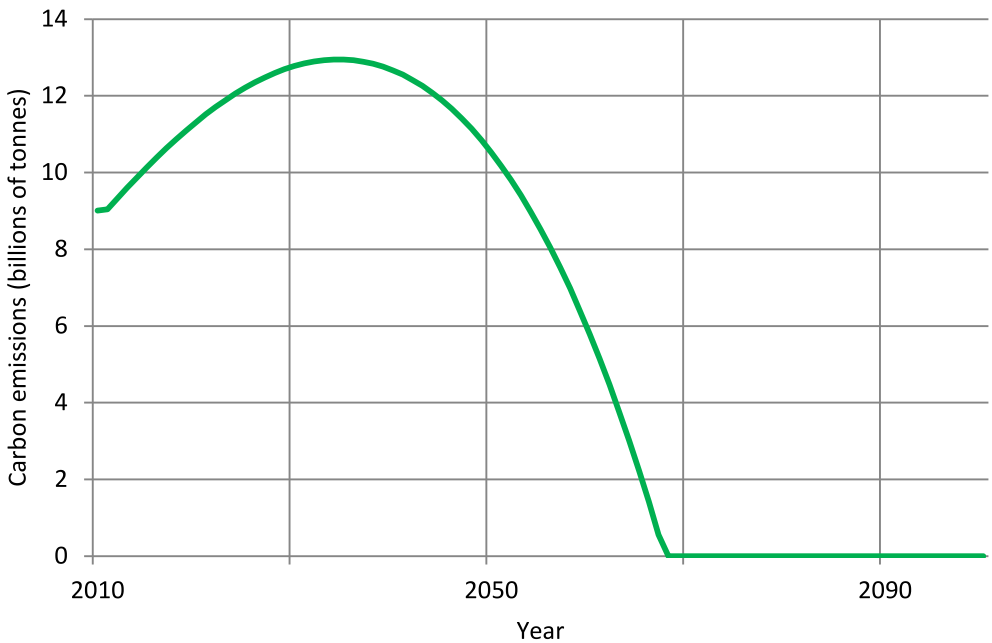
We choose an emissions reduction path for which government emissions regulation (the control rate, M_t) increases at a constant growth rate, m. We assume that in 2010, the emissions control rate is 9% and grows annually at a 4.267% growth rate (i.e., $M_t = M_{t-1}(1+m)$). The choice of emission control path here is relatively arbitrary and certainly not universally optimal.[21] The following equation describes how emissions control enters the model:

[21] The path is optimal (maximizes net present value) among all control paths that follow exponential growth assuming a 5% discount rate. Changing the discount rate affects which path is optimal.

$$E_t = (1 - M_t)\sigma_t Y_t$$

Given the assumed parameter values ($M_0 = 0.09$ and $m = 0.04267$), the annual emissions path is displayed in Figure 7. Emissions continue to increase but peak around 2035 and decline to zero by 2068. The emissions peak to reach the two degree target is almost half of the peak in the BAU scenario displayed in Figure 3A.

Figure 7. Predicted emissions path to achieve 2 Degree Limit



Source: authors' calculations.

Reducing emissions to zero and limiting temperature increase to 2 degrees reduces temperature damages on future generations; however, reducing emissions is not costless. To reflect the immediate cost of reducing emissions, we use the convex abatement cost function of Nordhaus and Sztorc (2013),

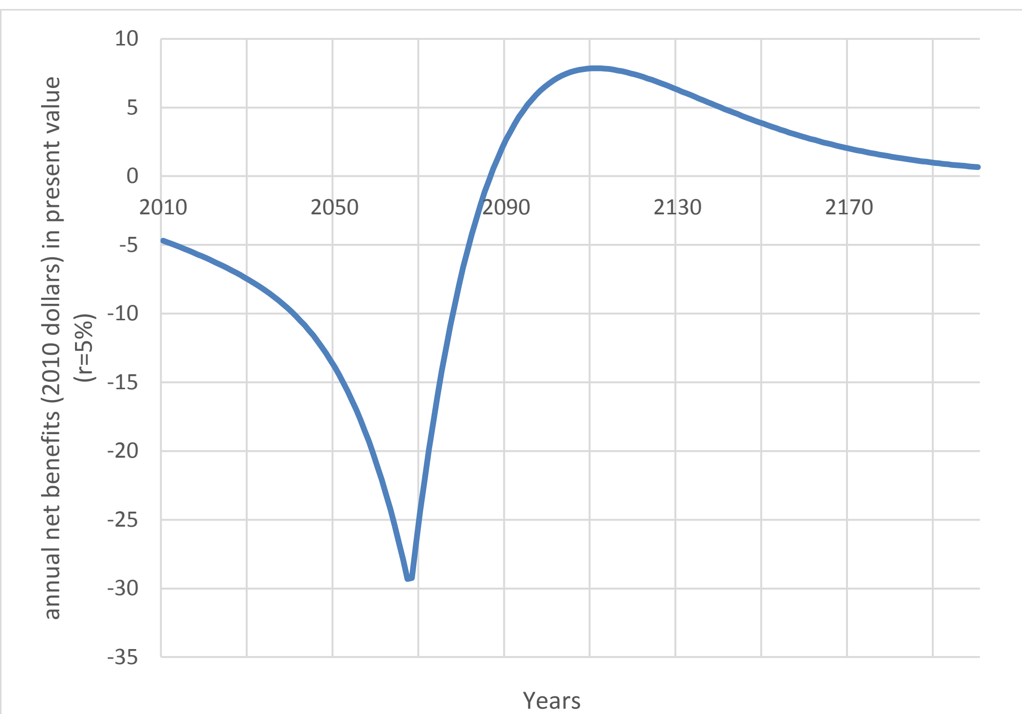
$$AC_t = \Omega_t M_t^2,$$

where $\Omega_0 = 0.06$ is an abatement cost coefficient that declines over time at the rate at which TFP grows. Total income per capita net of abatement cost in year t is

$$y_t = (1 - AC_t)D_t A_t k_t.$$

The reduction in per capita income of reducing emissions is difficult to show by just plotting the income path of BAU and the income path when the 2 degree limit is imposed. Instead Figure 8A shows the annual net benefits in present value of the 2 degree limit versus BAU over time assuming a 5% discount rate. The present value of total net benefits with a 5% discount rate is -$449 per capita. However, the net present value depends critically on the choice of discount rate. The annual net benefits of the 2 degree mitigation policy are displayed in Figure 8B, but in present values using the 1.4% discount rate selected by the Stern Review. In this case, the future annual net benefits do not limit to zero by 2200 and the present value of total net benefits is positive ($40,665 per capita). The Internal Rate of Return for limiting to 2 degrees is 4.08%.

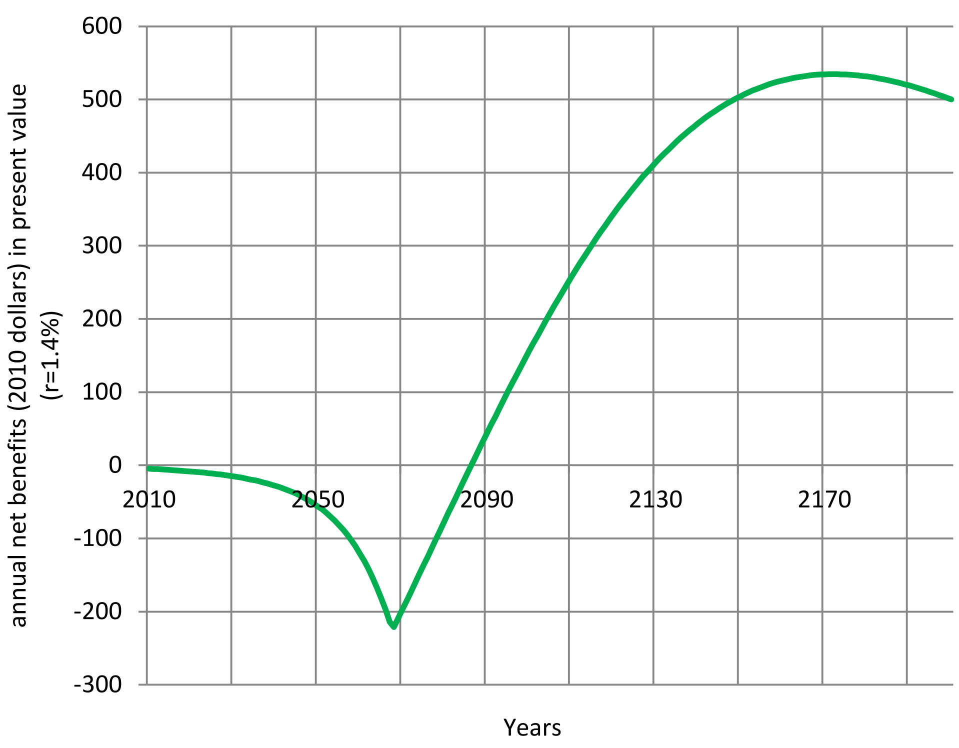
Figure 8A. Annual Net Benefits in Present Value using 5% Discount Rate



Source: authors' calculations.

What is interesting to highlight to students between the two figures, is that the discount rate determines if the 2 degree mitigation target is a potential Pareto improvement, but not a pure Pareto improvement. For both discount rate values, the annual net benefits still show the inter-generational trade-off in which the costs of emissions reductions are imposed on early generations and the benefits of lower temperature increases accrue to later generations. Unlike with intra-generational policy evaluation, in this case there is no potential mechanism for a future generation to compensate earlier generations to make them at least as well off. Mitigation policy is not asking the current generation to incur costs to benefit their children; it is asking them to incur costs to benefit their great- great-grandchildren.

Figure 8B. Annual Net Benefits in Present Value using 1.4% Discount Rate



Source: authors' calculations.

Classroom discussion: What factors determine the social discount rate? Does it make sense to have the social discount rate decline over time?

Classroom exercise: Read the article by Arrow (2007) and using an excel spreadsheet verify Arrow's cost benefit analysis on page 4-5. Discuss the implications of this finding.

Classroom exercise: Increase carbon emissions per person by 1 tonne in 2020 only and compare the net present value of income per person with the additional tonne and without. Repeat the exercise by adding an additional tonne of carbon in 2050 only. Discuss your findings.

7 Extensions and Concluding Remarks

This paper presents an extension of the Solow model that includes a simple climate model. To the best of our knowledge, we have developed the simplest Integrated Assessment Model. The simplicity of the model serves well for introducing the economics of climate change, which relies heavily on IAMs, to undergraduate students. Even though our model is very basic its predictions match well with Nordhaus' more complex DICE model.

In addition to introducing the model to students and highlighting the central intra-generational trade-off inherent in the climate change problem, we have also outlined two other classroom applications. The model can be used to teach the academic controversy over how damages from increased temperatures should enter the model and the implications of changing the standard assumptions on damages. The model is also very useful for teaching students about how economists approach evaluating the 2 degree Celsius target and the importance of the discount rate.

Although we have chosen to highlight only two specific applications of the model, many other learning applications are possible both in and out of the classroom. For example, students can be presented with the basic model and then asked to do their own simulations and to write up an essay on the economics of climate change. Students will observe that climate change will affect the standard of living of the world economy in the future and that climate change caused by humans is a global externality and as a result requires coordinated, corrective action by governments. They will also observe the intra-generational trade-offs involved, i.e., economic activity today inflicts a cost on future generation and that future generations will be richer than today's generation due to increases in productivity and the stabilization of global population. However, they will also observe that the richer future generations will not be as rich as they would have been without climate change. Since cost of action is absorbed by the current generation and the benefits of action accrue to future generations students can conduct a cost-benefit analysis and explore the importance of the discount rate. But future generations could be poorer than the current generation if temperature anomaly exceeds 3 Celsius and under a different damage function; different model parameters; when climate change affects the depreciation of capital and especially when temperature affects total factor productivity. In this case, the benefits of action relative to the costs in present value terms increase and stronger action is needed.

There are many extensions students can explore with the model beyond what is mentioned above (i.e., different damage impacts and a cost-benefit analysis). One example would be for students to conduct a sensitivity analysis by changing the parameters of the model and observing the impact on the standard of living. Secondly, in this paper we used the balanced growth path; however, simulations can be conducted according to the transitional growth path given by $k_{t+1} = k_t + sy_t - (\delta_K + g_{n,t})\, k_t$). Namely, the capital stock (per person) next period is equal to the capital stock per capita this period plus the difference between actual investment per person and the necessary investment that is needed to maintain the capital labor ratio constant. The new stock of capital is then channeled into the actual path of output per person through the production function $y_t = D_t A_t \, k_t^{\alpha}$. Using this approach students and instructors can explore the impact of sudden destructions of the capital stock or population due to tipping points. In the balanced growth model, the impact of sudden shocks are only for one period as the system returns to the steady state in the next period, but with the introduction of transitional dynamics the impact could last for over a decade until the system reaches a new steady state. This can be either done in-class led by the instructor or be given as an assignment so that students can compare and contrast the different paths of the economy.

Another option is to use the model to compute the social cost of carbon. We have found that students easily grasp the general idea of the social cost of carbon, but have difficulty understanding how the various estimates are calculated. Our model can be used to give students a hands-on demonstration of how the social cost of carbon is actually calculated. This is done easily by comparing the change in the net present value of steady state income per person under the base case carbon emissions path relative to a path when 1 additional tonne of carbon per person is added in a particular year. Students will see that the social cost of carbon increases if the tonne of carbon is released further into the future. Students will also immediately see why the choice of discount rate matters so crucially for the resulting social cost of carbon value.

A further application is to ask students to find parameters that are relevant to a particular nation and to examine the impact climate change will have on that particular nation. They can also examine the impact on economies that are emerging and growing faster relative to the industrial nations. For more advanced courses they could also change the economic model to an endogenous growth model and examine the impact of climate change.

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APPENDIX: Excel spreadsheet

It is straight forward to setup the spreadsheet and to show students the different future trajectories of the economic system. Students can also use this as a starting point to do their own explorations. In this section we show how to setup the model with all the damages. The excel spreadsheet will have one sheet for the parameter values and one to compute the steady state values. The “parameters” sheet has table 1. Column A is reserved for a description of the variables followed by column B for the symbol (not necessary) and column C with the assigned value. This is very easy to setup but important as all other sheets will be refereed to this sheet named “parameters”.

Table 1: Variables, symbols and values used for base case. “Parameters” sheet

		Column		
		A	B	C
Row	1	Description	Symbol	Value
	2	Capital's share of income	α	0.3
	3	Savings rate	s	0.25
	4	Depreciation rate	δ_0	0.1
	5	Impact of temperature on depreciation rate	δ_1	0.01
	6			
	7	Initial 2010 population (in billions)	L_0	6.838
	8	Initial 2010 population growth rate	$g_{L,0}$	0.023
	9	Parameter affecting population growth	δ_L	.052
	10			
	11	Initial 2010 total factor productivity	A_0	3.955
	12	Initial productivity growth rate	$g_{A,0}$	0.015
	13	Parameter affecting productivity growth	δ_A	0.011
	14	Temperature impact on productivity growth		0.001
	15			
	16	Initial world GDP (trillions of 2005 US $)	Y_0	63.69
	17	Initial world capital (trillion of 2005 US $s)	K_0	135
	18			
	19	Initial emission intensity	σ_0	0.549
	20	Initial 2010 growth of emissions intensity	$g_{\sigma,0}$	-0.01
	21	Parameter affecting emissions intensity growth	δ_σ	-0.0002
	23	Damage parameter	θ_1	0.002384
	24	Damage parameter	θ_2	2
	25	Damage parameter	θ_3	0.00000507
	26	Damage parameter	θ_4	6.754
	27			
	28	CCR	β	1.8
	29	Initial Carbon (billions of tonnes)		530

The next sheet will be the values of the various variables and is reported in Table 2 below. The first row are the variable names while the second row are the initial values of the variables. Column A list the years from 2010 to 2200. Column B records the growth rate of population over the years. The first value in cell B2 is the initial population growth rate of 2.3 percent in the sheet titled "parameters" under cell $C$8 (See table 1). The remaining values are calculated by having the population growth formula, $g_{L,t} = g_{L,t-1}/(1 + \delta_L)$, in each cell. This can be done by using the previous year population growth rate and dividing this by 1 + $C$9 from the "parameters" sheet. Column C computes the population level starting from the initial value of 6.838 billion obtained from parameters!$C$7 and multiplying this by the growth rate of that same year in column B. The growth rate of carbon intensity is listed in column D. The first value of the growth rate of carbon intensity is taken from table 1 as -0.01 in cell parameters!$C$20. The remaining values of this variable are generated from using the formula $g_{\sigma,t} = \frac{g_{\sigma,t-1}}{1+\delta_\sigma}$ where δ_σ Is taken from table 1, cell parameters!$C$21, while $g_{\sigma,t-1}$ is the previous value of the growth rate of carbon intensity (just above the current value). Carbon intensity in column E is then computed using the initial value for the first cell, found in the parameters sheet under $C$19, and then the remaining values are found using: $\sigma_t = \sigma_{t-1}(1 + g_{\sigma,t})$ where $g_{\sigma,t}$ is from column D and σ_{t-1} is the previous cell of carbon intensity. The next column, F, is steady state output per person but for the previous period. The first value is obtained as the initial value from table 1 by taking world GDP and dividing it by world population. The remaining values of output per person are all in period t-1. They are created using the previous steady state values of output per person from the last column of table 2. Carbon dioxide per person is easy to compute by multiplying carbon intensity in column E with output per person in column E. This is reported in column G. The next column H reports carbon emissions. Carbon emissions are C02 per person, column G, times the population value in column C and divided by 3.67. Column I is the cumulative carbon emissions obtained by adding to the current level of cumulative emissions, the carbon emissions in column H. Cumulative carbon emissions start from 530 billion tonnes of carbon found in $C$29 of the parameters sheet. Next is the temperature anomaly computed using the cumulative carbon emissions times 1.8 (cell parameters!$C$28) per 1000 billion of tonnes of carbon and thus divided by 1000 to bring it to the same units (i.e., billions tonnes of cumulative carbon). Next is column K where the growth rate of total factor productivity is listed. The first value in cell K2 is computed using parameters!$C$12-parameters!$C$14*J2. The other values are computed according to: parameters!$C$12/(1+parameters!$C$13)^(A3-$A$2)-parameters!$C$14*J column's corresponding value. Total factor productivity in column L is then computed as follows. The first value is taken from $C$11 in the parameters sheet. The rest of the values are obtained by taking the previous value in L column*(1+K3). The depreciation rate is computed in column N using parameters!$C$4+parameters!C$5*J column's corresponding temperature anomaly value. The damage function is obtained using the damage values in the parameters sheet and the corresponding temperature anomaly. The steady state capital labour ratio is then computed as $k_{ss,t} = \left[\frac{sA_tD_t}{\delta_K+g_{n,t}}\right]^{1/(1-\alpha)}$ where all values can be found in the previous columns and in the parameters sheet. For example P2 = ((parameters!$C$3*L2*O2)/(N2+B2))^(1/(1-parameters!$C$2)) etc. Similarly steady state output per person can be found in the last column and computed using the formula $y_{ss,t} = D_tA_t\, k_{ss,t}^{\alpha}$

Table 2: The path of the variables from 2010 to 2200

Row	Column															
	A	B	C	D	E	F	G	H	I	J	K	L	N	O	P	Q
1	Year	$g_{L,t}$	L_t	$g_{s,t}$	σ_t	y_{t-1}	CO_2/L_t	E_t	$C_0 + \sum^{t} E_i$	T_t	$g_{A,t}$	A_t	δ_k	D_t	k_t	y_t
2	2010	0.023	6.838	-0.010	0.549	9.314	5.313	9.899	530.000	0.954	0.014	3.955	0.110	0.998	17.683	9.343
3	2011	0.021	6.985	-0.010	0.544	9.343	5.078	9.664	539.899	0.972	0.014	4.011	0.110	0.998	18.224	9.560
4	2012	0.020	7.127	-0.010	0.538	9.560	5.144	9.990	549.563	0.989	0.014	4.067	0.110	0.998	18.768	9.779
5	2013	0.019	7.265	-0.010	0.533	9.779	5.209	10.312	559.553	1.007	0.014	4.123	0.110	0.998	19.313	9.999
6	2014	0.018	7.399	-0.010	0.527	9.999	5.273	10.630	569.865	1.026	0.014	4.180	0.110	0.997	19.856	10.219
7	2015	0.017	7.528	-0.010	0.522	10.219	5.335	10.944	580.495	1.045	0.013	4.236	0.110	0.997	20.399	10.440
8	2016	0.017	7.653	-0.010	0.517	10.440	5.396	11.253	591.439	1.065	0.013	4.292	0.111	0.997	20.940	10.661
9	2017	0.016	7.774	-0.010	0.512	10.661	5.455	11.556	602.692	1.085	0.013	4.349	0.111	0.997	21.480	10.883
10	2018	0.015	7.891	-0.010	0.507	10.883	5.513	11.853	614.247	1.106	0.013	4.405	0.111	0.997	22.017	11.104
11	2019	0.014	8.004	-0.010	0.501	11.104	5.568	12.144	626.100	1.127	0.013	4.461	0.111	0.997	22.550	11.325
12	2020	0.014	8.113	-0.010	0.496	11.325	5.623	12.429	638.244	1.149	0.013	4.517	0.111	0.997	23.081	11.547
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
	2197	0.000	10.524	-0.010	0.081	14.583	1.179	3.382	2714.698	4.886	-0.003	7.147	0.149	0.778	24.327	14.486
	2198	0.000	10.524	-0.010	0.080	14.486	1.159	3.324	2718.079	4.893	-0.003	7.126	0.149	0.777	24.156	14.390
	2199	0.000	10.524	-0.010	0.079	14.390	1.140	3.268	2721.403	4.899	-0.003	7.105	0.149	0.776	23.985	14.294
192	2200	0.000	10.524	-0.010	0.078	14.294	1.120	3.212	2724.671	4.904	-0.003	7.084	0.149	0.774	23.816	14.199